Hubbard 算符和 Pauli 算符之间的转换*

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摘 要 把用 Pauli 算符表示的哈密顿量转化成用 Hubbard 算符表示。利用 Hubbard 算符的归一性和正交性给出了 Hubbard 算符 和 Pauli 算符之间转化的运算公式,从而实现了哈密顿量表示方式的转化,得到了用 Hubbard 算符表示的自旋梯模型的哈密顿量。利用自旋梯模型的哈密顿量做为例子,完成了用 Pauli 算符表示的哈密顿量转化成用 Hubbard 算符表示。

关键词 哈密顿量;Hubbard 算符;Pauli 算符
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众所周知,物理学中哈密顿量是非常重要的,尤其是在量子力学中。在许多学科领域中都涉及到哈密顿量,例如:在可积模型中求散射矩阵 R[1],研究开边界条件[2]等等问题中,哈密顿量都起着重要的作用。但是在很多情况下哈密顿量是用不同的算符表达出来的,例如常用的算符有 Hubbard 算符[3]和 Pauli 算符[4],因此,研究这两种算符之间的转化就很重要。下面我们就研究 Pauli 算符和 Hubbard 算符之间的转化,我们以自旋梯模型的哈密顿量[4]做为例子来研究究 Pauli 算符和 Hubbard 算符之间的转化。

1 可积模型的哈密顿量及基矢

自旋梯模型的哈密顿量为:

$$H = \frac{1}{4}\sum_{j=1}^{N}(1+\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_{j+1})(1+\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_{j+1}) + \frac{1}{2}J\sum_{j=1}^{N}(\boldsymbol{\sigma}_j\cdot\boldsymbol{\tau}_j - 1) + \frac{1}{2}(J-\frac{1}{2})N \tag{1}$$

上式中 $\boldsymbol{\sigma}_j$ 和 τ_j 是作用在第 j 级梯的 Pauli 矩阵 J 是级梯间的耦合力。下面给出 Pauli 矩阵的表示

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix},\sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix},\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \tag{2}$$

我们定义产生算符 σ^+ 和湮灭算符 σ^-:

$$\sigma^+ = \frac{1}{2}(\sigma_x + i\sigma_y),\sigma^- = \frac{1}{2}(\sigma_x - i\sigma_y), \tag{3}$$

把方程(2)代如方程(3)得:

$$\sigma^+ = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix},\sigma^- = \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}, \tag{4}$$

首先,介绍运算中用到的一些符号和算符,假设有两种粒子 σ 和 τ,矩阵 σ 和矩阵 τ 代表两种粒子的 Pauli 算符,每种粒子的矩阵只作用在该粒子上,而对其它粒子不起作用。每种粒子只占据两种可能

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态中的一种态。算符 $X_j^{\alpha\beta} \equiv |\alpha_j\rangle\langle\beta_j|$ 代表 Hubbard 算符，$|\alpha_j\rangle$ 代表 Hilbert 空间第 j 个级梯的 Dirac 态并且同一级梯的 Dirac 态是正交的。态 $|\sigma,\tau\rangle$ 代表两种粒子的自旋态，态 $|\uparrow\rangle$ 代表自旋向上，态 $|\downarrow\rangle$ 代表自旋向下，算符的运算法则如下：

Pauli 算符表示的产生算符 σ^+ 和湮灭算符 σ^- 作用在 σ 粒子的自旋态上的运算法则为：

$$\sigma^+ |\downarrow\rangle = |\uparrow\rangle, \sigma^+ |\uparrow\rangle = 0. \sigma^- |\uparrow\rangle = |\downarrow\rangle, \sigma^- |\downarrow\rangle = 0. \tag{5}$$

Pauli 算符表示的产生算符 τ^+ 和湮灭算符 τ^- 作用在 τ 粒子的自旋态上的运算法则为：

$$\tau^+ |\downarrow\rangle = |\uparrow\rangle, \tau^+ |\uparrow\rangle = 0. \tau^- |\uparrow\rangle = |\downarrow\rangle, \tau^- |\downarrow\rangle = 0. \tag{6}$$

我们选择基矢为：

$$|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle), |1\rangle = |\uparrow,\uparrow\rangle ; |2\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle), |3\rangle = |\downarrow,\downarrow\rangle . \tag{7}$$

并且基矢是归一的，即

$$|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| = I \tag{8}$$

令 Pauli 算符 σ 作用在基矢上得：

$$\sigma^+ |0\rangle = -\frac{1}{\sqrt{2}}|1\rangle \tag{9}$$

$$\sigma^+ |1\rangle = 0 \tag{10}$$

$$\sigma^+ |2\rangle = \frac{1}{\sqrt{2}}|1\rangle \tag{11}$$

$$\sigma^+ |3\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle) \tag{12}$$

2 Hubbard 算符和 Pauli 算符之间的转换

本节主要介绍 Pauli 算符和 Hubbard 算符之间的转换关系。

产生算符 σ^+ 用 Hubbard 算符表示为：

$$\sigma^+ = \frac{1}{\sqrt{2}}(-X^{12} + X^{12} + X^{03} + X^{23}) \tag{13}$$

公式(1)中第一项可改写为公式(14)

$$\begin{aligned}(1 + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_{j+1})(1 + \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_{j+1}) = {} & 1 + \tau_j^z\tau_{j+1}^z + \sigma_j^z\sigma_{j+1}^z + \sigma_j^z\tau_j^z\sigma_{j+1}^z\tau_{j+1}^z + \\ & 2(\tau_j^+\tau_{j+1}^- + \tau_j^-\tau_{j+1}^+ + \sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+) + 2(\sigma_j^+\tau_j^z\sigma_{j+1}^-\tau_{j+1}^z + \sigma_j^-\tau_j^z\sigma_{j+1}^+\tau_{j+1}^z + \\ & \sigma_j^z\tau_j^-\sigma_{j+1}^z\tau_{j+1}^+ + \sigma_j^z\tau_j^z\sigma_{j+1}^z\tau_{j+1}^-) + 4(\sigma_j^+\tau_j^z\sigma_{j+1}^-\tau_{j+1}^- + \sigma_j^+\tau_j^-\sigma_{j+1}^-\tau_{j+1}^+ + \sigma_j^-\tau_j^+\sigma_{j+1}^+\tau_{j+1}^- + \sigma_j^-\tau_j^-\sigma_{j+1}^+\tau_{j+1}^+)\end{aligned} \tag{14}$$

式(1)中的第二项可表示为式(15)

$$\boldsymbol{\sigma}_j \cdot \boldsymbol{\tau}_j - 1 = -4X_j^{00} \tag{15}$$

因为 Hubbard 算符是正交的，故有

$$X^{\alpha\beta}X^{\alpha'\beta'} = \delta_{\beta\alpha'}X^{\alpha\beta'} \tag{16}$$

用 Hubbard 算符表示 Pauli 算符 σ 算符和 τ 算符为：

$$\begin{aligned}\tau_j^+\tau_{j+1}^- = \frac{1}{2}(& X_j^{10} + X_{j+1}^{01} + X_j^{10}X_{j+1}^{21} + X_j^{10}X_{j+1}^{32} - X_j^{10}X_{j+1}^{30} + X_j^{12}X_{j+1}^{01} + X_j^{12}X_{j+1}^{21} + X_j^{12}X_{j+1}^{32} - X_j^{12}X_{j+1}^{30} + \\ & X_j^{23}X_{j+1}^{01} + X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} - X_j^{23}X_{j+1}^{30} - X_j^{03}X_{j+1}^{01} - X_j^{03}X_{j+1}^{21} - X_j^{03}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30}\end{aligned} \tag{18}$$

$$\begin{aligned}\tau_j^-\tau_{j+1}^+ = \frac{1}{2}(& X_j^{01} + X_{j+1}^{10} + X_j^{01}X_{j+1}^{12} + X_j^{01}X_{j+1}^{23} - X_j^{01}X_{j+1}^{03} + X_j^{21}X_{j+1}^{10} + X_j^{21}X_{j+1}^{12} + X_j^{21}X_{j+1}^{23} - X_j^{21}X_{j+1}^{03} + \\ & X_j^{32}X_{j+1}^{10} + X_j^{32}X_{j+1}^{12} + X_j^{32}X_{j+1}^{23} - X_j^{32}X_{j+1}^{03} - X_j^{30}X_{j+1}^{10} - X_j^{30}X_{j+1}^{12} - X_j^{30}X_{j+1}^{23} + X_j^{30}X_{j+1}^{03}\end{aligned} \tag{19}$$

$$\sigma_j^+\sigma_{j+1}^- = \frac{1}{2}(X_j^{12} + X_{j+1}^{21} + X_j^{12}X_{j+1}^{32} + X_j^{12}X_{j+1}^{30} - X_j^{12}X_{j+1}^{01} + X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} + X_j^{23}X_{j+1}^{30} - X_j^{23}X_{j+1}^{01} +$$

$$X_j^{03}X_{j+1}^{21} + X_j^{03}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30} - X_j^{03}X_{j+1}^{01} - X_j^{10}X_{j+1}^{21} - X_j^{10}X_{j+1}^{32} - X_j^{10}X_{j+1}^{30} + X_j^{10}X_{j+1}^{01} \quad (20)$$

$$\sigma_j^-\sigma_{j+1}^+ = \frac{1}{2}(X_j^{21} + X_{j+1}^{12} + X_j^{21}X_{j+1}^{23} + X_j^{21}X_{j+1}^{03} - X_j^{21}X_{j+1}^{10} + X_j^{32}X_{j+1}^{12} + X_j^{32}X_{j+1}^{23} + X_j^{32}X_{j+1}^{03} - X_j^{32}X_{j+1}^{10} +$$
$$X_j^{30}X_{j+1}^{12} + X_j^{30}X_{j+1}^{23} + X_j^{30}X_{j+1}^{03} - X_j^{30}X_{j+1}^{10} - X_j^{10}X_{j+1}^{12} - X_j^{10}X_{j+1}^{23} - X_j^{10}X_{j+1}^{03} + X_j^{10}X_{j+1}^{10} \quad (21)$$

式(19)和式(21)的求和得：

$$\tau_j^+\tau_{j+1}^- + \sigma_j^+\sigma_{j+1}^- = (X_j^{10} + X_{j+1}^{01} - X_j^{10}X_{j+1}^{30} + X_j^{12}X_{j+1}^{21} + X_j^{12}X_{j+1}^{32} + X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} - X_j^{03}X_{j+1}^{01} + X_j^{03}X_{j+1}^{30} \quad (23)$$

式(20)和式(22)的求和得：

$$\tau_j^-\tau_{j+1}^+ + \sigma_j^-\sigma_{j+1}^+ = (X_j^{01} + X_{j+1}^{10} - X_j^{01}X_{j+1}^{03} + X_j^{21}X_{j+1}^{12} + X_j^{21}X_{j+1}^{23} + X_j^{32}X_{j+1}^{12} + X_j^{32}X_{j+1}^{23} - X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{03} \quad (24)$$

式(14)中第五项用 Hubbard 算符表示为：

$$2(\sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+ + \tau_j^+\tau_{j+1}^-\tau_j^-\tau_{j+1}^+ =$$
$$2[(X_j^{10} + X_{j+1}^{01} - X_j^{10}X_{j+1}^{30} + X_j^{12}X_{j+1}^{21} + X_j^{12}X_{j+1}^{32} + X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} - X_j^{03}X_{j+1}^{01} + X_j^{03}X_{j+1}^{30}) +$$
$$(X_j^{01} + X_{j+1}^{10} - X_j^{01}X_{j+1}^{03} + X_j^{21}X_{j+1}^{12} + X_j^{21}X_{j+1}^{23} + X_j^{32}X_{j+1}^{12} + X_j^{32}X_{j+1}^{23} - X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{03})] \quad (25)$$

Pauli **算符** σ^z **和** τ^z **用** Hubbard **算符表示为：**

$$\tau_j^z\tau_{j+1}^z = (X_j^{11} + X_{j+1}^{11} - X_j^{11}X_{j+1}^{33} - X_j^{11}X_{j+1}^{20} - X_j^{11}X_{j+1}^{02} - X_j^{33}X_{j+1}^{11} + X_j^{33}X_{j+1}^{33} + X_j^{33}X_{j+1}^{20} + X_j^{33}X_{j+1}^{02} -$$
$$X_j^{20}X_{j+1}^{11} + X_j^{20}X_{j+1}^{33} + X_j^{20}X_{j+1}^{20} + X_j^{20}X_{j+1}^{02} - X_j^{02}X_{j+1}^{11} + X_j^{02}X_{j+1}^{33} + X_j^{02}X_{j+1}^{20} + X_j^{02}X_{j+1}^{02} \quad (26)$$

$$\sigma_j^z\sigma_{j+1}^z = (X_j^{11} + X_{j+1}^{11} + X_j^{11}X_{j+1}^{02} + X_j^{11}X_{j+1}^{20} - X_j^{11}X_{j+1}^{33} + X_j^{02}X_{j+1}^{11} + X_j^{02}X_{j+1}^{02} + X_j^{02}X_{j+1}^{20} - X_j^{02}X_{j+1}^{33} +$$
$$X_j^{20}X_{j+1}^{11} + X_j^{20}X_{j+1}^{02} + X_j^{20}X_{j+1}^{20} - X_j^{20}X_{j+1}^{33} - X_j^{33}X_{j+1}^{11} - X_j^{33}X_{j+1}^{02} - X_j^{33}X_{j+1}^{20} + X_j^{33}X_{j+1}^{33} \quad (27)$$

对式(26)和式(27)求和，我们得式(14)的第二项和第三项用 Hubbard **算符表示为：**

$$\sigma_j^z\tau_j^z\sigma_{j+1}^z\tau_{j+1}^z = (X_j^{11} + X_{j+1}^{11} - X_j^{11}X_{j+1}^{33} - X_j^{33}X_{j+1}^{11} + X_j^{33}X_{j+1}^{33} + X_j^{20}X_{j+1}^{20} + X_j^{20}X_{j+1}^{02} + X_j^{02}X_{j+1}^{20} + X_j^{02}X_{j+1}^{02}) \quad (28)$$

我们得式(14)的第四项用 Hubbard **算符表示为：**

$$\sigma_j^z\tau_j^z\sigma_{j+1}^z\tau_{j+1}^z = (X_j^{11} + X_{j+1}^{11} - X_j^{11}X_{j+1}^{00} - X_j^{11}X_{j+1}^{22} + X_j^{11}X_{j+1}^{33} - X_j^{00}X_{j+1}^{11} + X_j^{00}X_{j+1}^{00} + X_j^{00}X_{j+1}^{22} - X_j^{00}X_{j+1}^{33} -$$
$$X_j^{22}X_{j+1}^{11} + X_j^{22}X_{j+1}^{00} + X_j^{22}X_{j+1}^{22} - X_j^{22}X_{j+1}^{33} + X_j^{33}X_{j+1}^{11} - X_j^{33}X_{j+1}^{00} - X_j^{33}X_{j+1}^{22} + X_j^{33}X_{j+1}^{33} \quad (29)$$

Pauli **算符** σ^1 **和** τ^1 $(1 = +, -, z)$ **的积用** Hubbard **算符表示为：**

$$\sigma_j^+\tau_j^z\sigma_{j+1}^-\tau_{j+1}^z = \frac{1}{2}(- X_j^{10}X_{j+1}^{21} + X_j^{10}X_{j+1}^{30} + X_j^{10}X_{j+1}^{32} + X_j^{10}X_{j+1}^{01} - X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{30} + X_j^{23}X_{j+1}^{32} + X_j^{23}X_{j+1}^{01} -$$
$$X_j^{03}X_{j+1}^{21} + X_j^{03}X_{j+1}^{30} + X_j^{03}X_{j+1}^{32} + X_j^{03}X_{j+1}^{01} + X_j^{12}X_{j+1}^{21} - X_j^{12}X_{j+1}^{30} - X_j^{12}X_{j+1}^{32} - X_j^{12}X_{j+1}^{01} \quad (30)$$

$$\sigma_j^-\tau_j^z\sigma_{j+1}^+\tau_{j+1}^z = \frac{1}{2}(- X_j^{21}X_{j+1}^{10} - X_j^{21}X_{j+1}^{23} - X_j^{21}X_{j+1}^{03} + X_j^{21}X_{j+1}^{12} + X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{23} + X_j^{30}X_{j+1}^{03} - X_j^{30}X_{j+1}^{12} +$$
$$X_j^{32}X_{j+1}^{10} + X_j^{32}X_{j+1}^{23} + X_j^{32}X_{j+1}^{03} - X_j^{32}X_{j+1}^{12} + X_j^{01}X_{j+1}^{10} + X_j^{01}X_{j+1}^{23} + X_j^{01}X_{j+1}^{03} - X_j^{01}X_{j+1}^{12} \quad (31)$$

$$\sigma_j^z\tau_j^+\sigma_{j+1}^z\tau_{j+1}^- = \frac{1}{\sqrt{2}}(X_j^{10} + X_j^{12} + X_j^{03} - X_j^{23}) \frac{1}{\sqrt{2}}(X_{j+1}^{21} X_{j+1}^{01} - X_{j+1}^{32} + X_{j+1}^{30})$$
$$= \frac{1}{2}(X_j^{10}X_{j+1}^{21} + X_j^{10}X_{j+1}^{01} - X_j^{10}X_{j+1}^{32} + X_j^{10}X_{j+1}^{30} + X_j^{12}X_{j+1}^{21} + X_j^{12}X_{j+1}^{01} - X_j^{12}X_{j+1}^{32} + X_j^{12}X_{j+1}^{30} +$$
$$X_j^{03}X_{j+1}^{21} + X_j^{03}X_{j+1}^{01} - X_j^{03}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30} - X_j^{23}X_{j+1}^{21} - X_j^{23}X_{j+1}^{01} + X_j^{23}X_{j+1}^{32} - X_j^{23}X_{j+1}^{30}) \quad (32)$$

$$\sigma_j^z\tau_j^-\sigma_{j+1}^z\tau_{j+1}^+ = \frac{1}{2}(X_j^{21} + X_{j+1}^{10} + X_j^{21}X_{j+1}^{12} + X_j^{21}X_{j+1}^{03} - X_j^{21}X_{j+1}^{23} + X_j^{01}X_{j+1}^{10} + X_j^{01}X_{j+1}^{12} + X_j^{01}X_{j+1}^{03} - X_j^{01}X_{j+1}^{23} + + X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{12} + X_j^{30}X_{j+1}^{03} - X_j^{30}X_{j+1}^{23} - X_j^{32}X_{j+1}^{10} - X_j^{32}X_{j+1}^{12} - X_j^{32}X_{j+1}^{03} + X_j^{32}X_{j+1}^{23} \tag{33}$$

式(30)和(32)的和为：

$$\sigma_j^+\tau_j^z\sigma_{j+1}^-\tau_{j+1}^z + \sigma_j^z\tau_j^+\sigma_{j+1}^z\tau_{j+1}^- = (X_j^{10}X_{j+1}^{30} + X_j^{10}X_{j+1}^{01} - X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30} + X_j^{03}X_{j+1}^{01} + X_j^{12}X_{j+1}^{21} - X_j^{12}X_{j+1}^{32}) \tag{34}$$

式(31)和(33)的和为：

$$\sigma_j^-\tau_j^z\sigma_{j+1}^+\tau_{j+1}^z + \sigma_j^z\tau_j^-\sigma_{j+1}^z\tau_{j+1}^+ = (-X_j^{21}X_{j+1}^{23} + X_j^{21}X_{j+1}^{12} + X_j^{01}X_{j+1}^{10} + X_j^{01}X_{j+1}^{03} + X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{03} + X_j^{32}X_{j+1}^{23} - X_j^{32}X_{j+1}^{12}) \tag{35}$$

由式(34)和式(35)我们得式(14)的第六项用 Hubbard 算符表示为：

$$\sigma_j^+\tau_j^z\sigma_{j+1}^-\tau_{j+1}^z + \sigma_j^z\tau_j^+\sigma_{j+1}^z\tau_{j+1}^- + \sigma_j^-\tau_j^z\sigma_{j+1}^+\tau_{j+1}^z + \sigma_j^z\tau_j^-\sigma_{j+1}^z\tau_{j+1}^+ = (X_j^{10}X_{j+1}^{30} + X_j^{10}X_{j+1}^{01} - X_j^{23}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30} + X_j^{03}X_{j+1}^{01} + X_j^{12}X_{j+1}^{21} - X_j^{12}X_{j+1}^{32}) - X_j^{21}X_{j+1}^{23} + X_j^{21}X_{j+1}^{12} + X_j^{01}X_{j+1}^{10} + X_j^{01}X_{j+1}^{03} + X_j^{30}X_{j+1}^{10} + X_j^{30}X_{j+1}^{03} + X_j^{32}X_{j+1}^{23} - X_j^{32}X_{j+1}^{12}) \tag{36}$$

Pauli 算符 σ^i 和 τ^i ($i = +, -$)的积用 Hubbard 算符表示为：

$$\sigma_j^+\tau_j^+ = X_j^{13} \tag{37}$$

$$\sigma_j^-\tau_j^- = X_j^{31} \tag{38}$$

$$\sigma_j^+\tau_j^- = \frac{1}{2}(X_j^{32} - X_j^{20} + X_j^{02} - X_j^{00}) \tag{39}$$

$$\sigma_j^-\tau_j^+ = \frac{1}{2}(X_j^{20} + X_j^{22} - X_j^{00} - X_j^{02}) \tag{40}$$

$$\sigma_j^z\tau_j^+ = \frac{1}{\sqrt{2}}(X_j^{10} + X_j^{12} + X_j^{03} - X_j^{23}) \tag{41}$$

$$\sigma_j^z\tau_j^- = \frac{1}{\sqrt{2}}(X_j^{21} + X_j^{01} - X_j^{32} + X_j^{30}) \tag{42}$$

$$\tau_j^z\sigma_j^+ = \frac{1}{\sqrt{2}}(X_j^{12} - X_j^{10} - X_j^{23} - X_j^{03}) \tag{43}$$

$$\tau_j^z\sigma_j^- = \frac{1}{\sqrt{2}}(X_j^{21} - X_j^{01} - X_j^{32} - X_j^{30}) \tag{44}$$

$$\sigma_j^+\tau_j^z = \frac{1}{\sqrt{2}}(-X_j^{10} - X_j^{23} - X_j^{03} + X_j^{12}) \tag{45}$$

$$\sigma_j^-\tau_j^z = \frac{1}{\sqrt{2}}(X_j^{21} - X_j^{30} - X_j^{32} - X_j^{01}) \tag{46}$$

$$\sigma_j^z\tau_j^z = (X_j^{11} - X_j^{00} - X_j^{22} + X_j^{33}) \tag{47}$$

$$\sigma_j^+\tau_j^+\sigma_{j+1}^-\tau_{j+1}^- = X_j^{13}X_{j+1}^{31} \tag{48}$$

$$\sigma_j^+\tau_j^-\sigma_{j+1}^-\tau_{j+1}^+ = \frac{1}{4}(X_j^{02}X_{j+1}^{20} + X_j^{02}X_{j+1}^{22} - X_j^{02}X_{j+1}^{00} - X_j^{02}X_{j+1}^{02} + X_j^{22}X_{j+1}^{20} + X_j^{22}X_{j+1}^{22} - X_j^{22}X_{j+1}^{00} - X_j^{22}X_{j+1}^{02} - X_j^{00}X_{j+1}^{20} - X_j^{00}X_{j+1}^{22} + X_j^{00}X_{j+1}^{00} + X_j^{00}X_{j+1}^{02} - X_j^{20}X_{j+1}^{20} - X_j^{20}X_{j+1}^{22} + X_j^{20}X_{j+1}^{00} + X_j^{20}X_{j+1}^{02}) \tag{49}$$

$$\sigma_j^-\tau_j^+\sigma_{j+1}^+\tau_{j+1}^- = \frac{1}{4}(X_j^{20}X_{j+1}^{02} + X_j^{20}X_{j+1}^{22} - X_j^{20}X_{j+1}^{00} - X_j^{20}X_{j+1}^{02} + X_j^{22}X_{j+1}^{02} + X_j^{22}X_{j+1}^{22} - X_j^{22}X_{j+1}^{00} - X_j^{22}X_{j+1}^{20} - X_j^{00}X_{j+1}^{02} - X_j^{00}X_{j+1}^{22} + X_j^{00}X_{j+1}^{00} + X_j^{00}X_{j+1}^{20} - X_j^{02}X_{j+1}^{02} - X_j^{02}X_{j+1}^{22} + X_j^{02}X_{j+1}^{00} + X_j^{02}X_{j+1}^{20}) \tag{50}$$

$$\sigma_j^-\tau_j^-\sigma_{j+1}^+\tau_{j+1}^+ = X_j^{31}X_{j+1}^{13} \tag{51}$$

式(50)和式(51)求和得：

$$\sigma_j^+\tau_j^-\sigma_{j+1}^-\tau_{j+1}^+ + \sigma_j^-\tau_j^+\sigma_{j+1}^+\tau_{j+1}^- =$$
$$\frac{1}{2}(X_j^{02}X_{j+1}^{20} - X_j^{02}X_{j+1}^{02} + X_j^{22}X_{j+1}^{22} - X_j^{22}X_{j+1}^{00} - X_j^{00}X_{j+1}^{22} + X_j^{00}X_{j+1}^{00} - X_j^{20}X_{j+1}^{20} + X_j^{20}X_{j+1}^{02}) \quad (52)$$

由式(48)、(51)和式(52)我们得式(14)的第七项用 Hubbard 算符表示为:

$$\sigma_j^+\tau_j^+\sigma_{j+1}^-\tau_{j+1}^- + \sigma_j^-\tau_j^-\sigma_{j+1}^+\tau_{j+1}^+ ++ \sigma_j^+\tau_j^-\sigma_{j+1}^-\tau_{j+1}^+ + \sigma_j^-\tau_j^+\sigma_{j+1}^+\tau_{j+1}^- =$$
$$X_j^{13}X_{j+1}^{31} + X_j^{31}X_{j+1}^{13} + \frac{1}{2}(X_j^{02}X_{j+1}^{20} - X_j^{02}X_{j+1}^{02} + X_j^{22}X_{j+1}^{22} - X_j^{22}X_{j+1}^{00} - X_j^{00}X_{j+1}^{22} + X_j^{00}X_{j+1}^{00} - X_j^{20}X_{j+1}^{20} + X_j^{20}X_{j+1}^{02}) \quad (53)$$

故我们得式(14)用 Hubbard 算符表示为:

$$(1 + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_{j+1})(1 + \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_{j+1}) =$$
$$1 + 4(X_j^{10}X_{j+1}^{01} + X_j^{12}X_{j+1}^{21} + X_j^{23}X_{j+1}^{32} + X_j^{03}X_{j+1}^{30} + X_j^{01}X_{j+1}^{10} + X_j^{32}X_{j+1}^{23} + X_j^{30}X_{j+1}^{03}) + 4(X_j^{20}X_{j+1}^{02} + X_j^{02}X_{j+1}^{20} + X_j^{21}X_{j+1}^{12}) + 3(X_j^{00}X_{j+1}^{00} + X_j^{11}X_{j+1}^{11} + X_j^{22}X_{j+1}^{22} + X_j^{33}X_{j+1}^{33}) - X_j^{00}X_{j+1}^{11} - X_j^{00}X_{j+1}^{22} - X_j^{00}X_j^{33} - X_j^{11}X_{j+1}^{00} - X_j^{11}X_{j+1}^{22} - X_j^{11}X_{j+1}^{33} - X_j^{22}X_{j+1}^{00} - X_j^{22}X_{j+1}^{11} - X_j^{22}X_{j+1}^{33} - X_j^{33}X_{j+1}^{00} - X_j^{33}X_{j+1}^{11} - X_j^{33}X_{j+1}^{22} \quad (54)$$

同时,归一性可等价与下式:

$$1 = (X^{00} + X^{11} + X^{22} + X^{33})_j(X^{00} + X^{11} + X^{22} + X^{33})_j =$$
$$X_j^{22}X_{j+1}^{00} + X_j^{22}X_{j+1}^{11} + X_j^{22}X_{j+1}^{22} + X_j^{22}X_{j+1}^{33} + X_j^{33}X_{j+1}^{00} + X_j^{33}X_{j+1}^{11} + X_j^{33}X_{j+1}^{22} + X_j^{33}X_{j+1}^{33} \quad (55)$$

把式(55)代入式(54)得:

$$(1 + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_{(j+1)}(1 + \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_{j+1}) = 4\sum_{\alpha,\beta=0}^{3} X_j^{\alpha\beta}X_{j+1}^{\beta\alpha} \quad (56)$$

由以上运算后我们得哈密顿量(1)用 Hubbard 算符表示为:

$$H = \frac{1}{4} \times 4\sum_{j=1}^{N}\sum_{\alpha,\beta=0}^{3} X_j^{\alpha\beta}X_{j+1}^{\beta\alpha} + \frac{1}{2}J\sum_{j=1}^{N}(-4X_j^{00}) + \frac{1}{2}(J - \frac{1}{2})N$$
$$= \sum_{j=1}^{N}\sum_{\alpha,\beta=0}^{3} X_j^{\alpha\beta}X_{j+1}^{\beta\alpha} - 2J\sum_{j=0}^{N} X_j^{00} + \frac{1}{2}(J - \frac{1}{2})N \quad (57)$$

即

$$H = \sum_{j=1}^{N}\sum_{\alpha,\beta=0}^{3} X_j^{\alpha\beta}X_{j+1}^{\beta\alpha} - 2J\sum_{j=1}^{N} X_j^{00} + \frac{1}{2}(J - \frac{1}{2})N \quad (58)$$

这样我们就完成了两种算符之间的转化。

3 结论

利用自旋梯可积模型的哈密顿量,介绍了 Pauli 算符和 Hubbard 算符之间的转化。利用两算符之间的关系我们也可进行其它哈密顿量在两种算符之间的转化,有了这种转化关系将对我们的工作带来许多方便。

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Dissipation Controller Design for a Class of Neutral Delay Systems

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Abstract: This paper focuses on a class of neutral delay systems. We are concerned with the design of dissipative static state feedback controller such that the closed – loop system is strictly dissipative. Sufficient conditions for the existence of the dissipative controller is obtained by using a linear matrix inequality (LMI) approach. Furthermore, we provide a procedure of constructing such controller from the solutions of LMIs. The findings shows that the solvability of dissipative controller design problem is implied by the feasibility of LMIs. It contains control rate and passive control supply rate as its special cases.

Keywords: nonlinear uncertain, time – Delay system, dissipation

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The Transformation Between Hubbard Operators and Pauli Matrices

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Abstract: Aiming at the transformation of Hamiltonian from Pauli matrices to Hubbard operators, the authors use the unity and orthogonal of Hubbard operators to give the transforming formula of Pauli matrices and Hubbard operators, by which, they realize the change from Pauli matrices to Hubbard operators. The result has been obtained that the Hamiltonian expressed by Hubbard operators of the spin – ladder model.

Keywords: Hamiltonian; Hubbard operators; Pauli matrices.