极坐标系下应力偏张量主值和主应力的表达式*

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摘　要:通过极坐标与直角坐标的关系,用不同于塑性力学教材的另一种方法推导得出应力偏张量主值和主应力在极坐标下的表达式,并与塑性力学教材给出的公式进行比较,对在极坐标系下的解题提供简化过程,为理解结构的某些力学性质奠定理论基础。

关键词:应力偏张量;主应力;极坐标

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在现有的塑性力学的教材中,大部分公式都是在直角坐标系中得到的,然后利用极坐标系与直角坐标的关系来得到极坐标下的公式。然而在实际的应用中,使用极坐标系下的公式具有一定的普遍性。往往在某些时候使用极坐标系下的公式来解题可以简化计算过程,也可以直接说明结构的某些力学性质,为解释一些力学现象提供方便。本文通过一种与塑性力学教材所不同的推导方法得出极坐标下应力偏张量主值和主应力的表达式,对理解极坐标系下的公式提供帮助。

1　理论分析

在 π 平面内,某一点的直角坐标系下的 x,y 与极坐标系下的 r,θ_σ 与应力偏量主值 s_1,s_2,s_3 间存在如下关系[1]:

$$x = \frac{\sqrt{2}}{2}(s_1 - s_3) \tag{1}$$

$$y = \frac{2s_2 - s_1 - s_3}{\sqrt{6}} \tag{2}$$

$$r_\sigma = \sqrt{x^2 + y^2} = \sqrt{2J2} \tag{3}$$

$$\tan\theta_\sigma = \frac{y}{x} = \frac{1}{\sqrt{3}}\frac{2s_2 - s_1 - s_3}{\sqrt{3}s_1 - s_3} \tag{4}$$

当规定 $s_1 \geq s_2 \geq s_3$ 时,有 $-\frac{\pi}{6} \leq \theta_\sigma \leq \frac{k}{6}$。

下面用 r,θ_σ 来表示 s_1,s_2,s_3:

由(1)式得:$s_1 - s_3 = \sqrt{2}x = \sqrt{2}r_\sigma\cos\theta_\sigma$　　(5)

又因为应力偏张量主值间存在如下关系:$s_1 + s_2 + s_3$;故:

$$s_1 + s_3 = (s_1 + s_3) - \frac{2}{3}(s_1 + s_2 + s_3) = \frac{1}{3}(s_1 - 2s_2 + s_3) = -\frac{\sqrt{6}}{3}y = -\frac{\sqrt{6}}{3}r_\sigma\sin\theta_\sigma \tag{6}$$

联立(5)式和(6)式,可得:

$$s_1 = \frac{1}{2}r_\sigma(\sqrt{2}\cos\theta_\sigma - \frac{\sqrt{6}}{3}\sin\theta_\sigma) = \sqrt{\frac{2}{3}}\sin(\theta_\sigma + \frac{2}{3}\pi)$$

$$s_3 = \frac{1}{2}r_\sigma(-\sqrt{2}\cos\theta_\sigma - \frac{\sqrt{6}}{3}\sin\theta_\sigma) = \sqrt{\frac{2}{3}}r_\sigma\sin(\theta_\sigma - \frac{2}{3}\pi) \tag{7}$$

$$s_2 = -(s_1 + s_3) = \sqrt{\frac{2}{3}}r_\sigma\sin\theta_\sigma$$

有了应力偏张量主值的表达式,易得到主应力的表达式:

$$\sigma_1 = s_1 + \sigma_m = \frac{1}{2}r_\sigma(\sqrt{2}\cos\theta_\sigma - \frac{\sqrt{6}}{3}\sin\theta_\sigma) + \sigma_m =$$

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$$\sqrt{\frac{2}{3}}\sin(\theta_\sigma+\frac{2}{3}\pi)+\sigma_m;$$

$$\sigma_2=s_2+\sigma_m=-(s_1+s_3)+\sigma_m=\sqrt{\frac{2}{3}}r_\sigma\sin\theta_\sigma+\sigma_m;$$

$$\sigma_3=s_3+\sigma_m=\frac{1}{2}r_\sigma(-\sqrt{2}\cos\theta_\sigma-\frac{\sqrt{6}}{3}\sin\theta_\sigma)+\sigma_m=\sqrt{\frac{2}{3}}r_\sigma\sin(\theta_\sigma-\frac{2}{3}\pi)+\sigma_m;$$

当 r_σ 和 lode 参数 μ_σ 为已知时,可利用 μ_σ 与 θ_σ 间的关系:

$$\theta_\sigma=\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})$$

得到用 r_σ 和 lode 参数 μ_σ 表示的应力偏量主值和主应力:

$$s_1=\frac{1}{2}r_\sigma(\sqrt{2}\cos[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})]-\frac{\sqrt{6}}{3}\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}}])=\sqrt{\frac{2}{3}}\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})+\frac{2}{3}\pi];$$

$$s_2=-(s_1+s_3)=\sqrt{\frac{2}{3}}r_\sigma\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})];$$

$$s_3=\frac{1}{2}r_\sigma\{-\sqrt{2}\cos[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})-\frac{\sqrt{6}}{3}\sin(\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})]\}=\sqrt{\frac{2}{3}}\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})-\frac{2}{3}\pi];$$

$$\sigma_1=s_1+\sigma_m=\sqrt{\frac{2}{3}}\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})+\frac{2}{3}\pi]+\sigma_m;$$

$$\sigma_2=s_2+\sigma_m=-(s_1+s_3)+\sigma_m=\sqrt{\frac{2}{3}}r_\sigma\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})+\frac{2}{3}\pi]+\sigma_m;$$

$$\sigma_3=s_3+\sigma_m=\frac{1}{2}r_\sigma(-\sqrt{2}\cos\theta_\sigma-\frac{\sqrt{6}}{3}\sin\theta_\sigma)+\sigma_m=\sqrt{\frac{2}{3}}r_\sigma\sin[\tan^{-1}(\frac{\mu_\sigma}{\sqrt{3}})-\frac{2}{3}\pi]+\sigma_m;$$

对于应力特征主角 ω_σ[2] 与 θ_σ(应力偏向量与 x 轴的夹角)[1]的关系:

由文献[2]知:
$$\cos 3\omega_\sigma=-\frac{3\sqrt{3}}{2}\frac{J'_3}{J'_2\sqrt{J'2}} \quad (8)$$

式中:
$$0\leqslant\omega_\sigma\leqslant\frac{\pi}{3} \quad (9)$$

考虑(7)式,有:

$$J'_3=s_1s_2s_3=-\left(\frac{4}{3}J'_2\right)^{\frac{3}{2}}\frac{\sin 3\theta_\sigma}{4}$$

即:
$$\sin 3\theta_\sigma=\frac{-3\sqrt{3}}{2}\frac{J_3}{J'_2\sqrt{J'_2}} \quad (10)$$

式中,
$$-\frac{\pi}{6}\leqslant\theta_\sigma\leqslant\frac{\pi}{6} \quad (11)$$

由(8)~(11)式,知 ω_σ 与 θ_σ 间的关系为:

$$\theta_\sigma=\omega_\sigma-\frac{\pi}{6} \quad (12)$$

2　结论

本文通过另一种推导方法得到应力偏张量主值以及主应力在极坐标系下的表达式,进而证明了该表达式的正确性,简化了在极坐标系下解题的过程,对理解一些力学现象的含义提供理论基础。

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Expression of main stress and main stress partial tensor in polar coordinate

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Abstract: Based on the relation of polar coordinate and right - angle coordinate, a new method is proposed to induce the expression of the main stress and the main stress partial tensor in polar coordinate. The induced expression is compared with the one from the plastic mechanics teaching materials, which can help us resolve the problems in the polar coordinate simply and understand the characters of some mechanics significations.

Keywords: polar coordinate; right - angle coordinate; main stress; main stress partial tensor

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Analysis of DDT in soil sample by direct sample introduction

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Abstract: This paper studies, a new analytical method for the pesticide residue in soil sample. This method is based on a novel direct sample introduction instrument called ChromatProbe that enables extraction of free soil sample into GC. The authors keep the soil sample in the balancing chamber for some time, then put the sample into the tiny sample bottle which was sent into the GC and obtained the quantity of DDT in the soil.

Keywords: pesticide residues; GC; soil Direct Sample Introduction