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# 远端剪切作用下正交各向异性介质中椭圆夹杂的弹性场分析

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摘要:求解一类正交各向异性介质中平面椭圆夹杂在远端作用与椭圆主轴呈任意角度均匀剪切力情况下,内受非弹性特征应变引起的弹性场。采用各向异性平面问题的复变函数解法,结合保角变换方法,将远端剪切作用转化为在基体内边界上的初始应变,根据最小应变能原理,获得夹杂/基体系统弹性应力和应变场的封闭形式解析解。

关键词:非弹性特征应变;正交各向异性;保角变换;椭圆夹杂
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复合材料及其结构已在工程中得到广泛应用,然而材料中存在的各种微结构会严重影响材料的整体性能。确定关于这些微结构的弹性力学场对于弄清材料的断裂损伤和疲劳失效行为具有重要意义。人们对具有正交各向异性的材料与结构已经进行了大量的研究,Eshelby 采用特征应变方法对单个椭圆(球)夹杂问题进行了开创性的研究[1],Mura 的专著集中描述了该方法可以广泛而有效地用于解决各种夹杂问题[2]。此外,基于 Lekhnitskii 的经典工作[3]和 Stroh 理论[4,5],更多研究工作关注在各种外力作用下求解各向异性材料的孔口问题。

对于包含异质体夹杂物的材料而言,当环境温度发生变化时,夹杂与基体之间将产生相互作用[6,7]。由于在平面问题中椭圆可以较好模拟夹杂形态,本文考虑远端存在剪切作用下,正交各向异性介质中包含单个椭圆异质体夹杂受到非弹性特征应变作用而产生的弹性场问题。采用各向异性复变函数方法,结合保角变换,先将远端剪切作用转化为在基体内边界上的初始应变,然后利用4个表征平衡边界的待定参数分别表示夹杂和基体各自的弹性应变能的表达式。根据最小应变能原理,得到这些参数的解析表达式,从而得到应力场和应变场的封闭形式解析解。通过检验椭圆边界上的法向正应力和剪应力的连续条件来表明结果的正确性。

## 1 问题的描述

将考虑如图1所示的正交各向异性介质中包含一个尺寸相对很小的椭圆异质体夹杂,在远离夹杂的边缘部位上分布着密度为切应力,可以认为是作用在无限原处,并且切应力的方向与椭圆

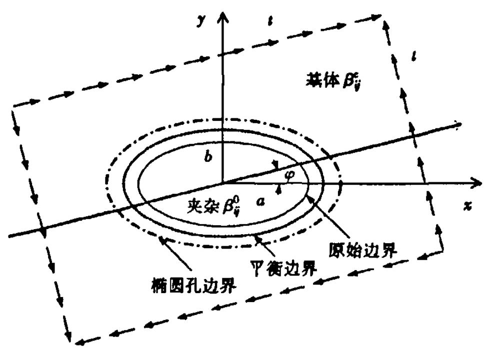


图1 远端剪切作用下的椭圆夹杂变形示意图
Fig.1 Schematic of an elliptical inhomogeneity in the matrix under shear

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夹杂的长主轴 $x$ 轴之间存在任意夹角 $\varphi$。椭圆夹杂沿坐标 $x$ 轴和 $y$ 轴的半轴长分别为 $a$ 和 $b$,半轴比为 $R=b/a$。

若基体内含的是椭圆空洞,根据 Lekhnitskii 的理论,在无穷远处的外力作用,不会在椭圆孔内边界上产生正应力和切应力,但会在基体中引起"初始"位移

$$u^c - \varepsilon_a^c x + \gamma_a^c y, v^c = \varepsilon_b^c y + \gamma_b^c x \tag{1}$$

式中 $\varepsilon_a^c$、$\varepsilon_b^c$、$\gamma_a^c$、$\gamma_b^c$ 为"初始"应变,以下与基体有关的物理量加注上标"$c$"。

但是,基体内实际含的是异质夹杂体,同时夹杂内还存在非弹性特征应变 $\varepsilon_a^0$、$\varepsilon_b^0$、$\gamma_a^0$、$\gamma_b^0$。所以基体——夹杂边界最终变形到由 $\varepsilon_1$、$\varepsilon_2$、$\gamma_1$、$\gamma_2$ 所表征的平衡边界,其对应的位移为

$$u = \varepsilon_1 x + \gamma_1 y, v = \varepsilon_2 y + \gamma_2 x \tag{2}$$

在这过程中,由特征应变引起的基体位移为

$$u_x^c = D_0 x + I_1 y, \quad v_y^c = E_0 y + I_2 x \tag{3}$$

式中:

$$D_0 = \varepsilon_1 - \varepsilon_a^c, \quad I_1 = \gamma_- \gamma_a^c,$$

$$E_0 = \varepsilon_2 - \varepsilon_b^c, \quad I_2 = \gamma_2 - \gamma_b^c$$

问题即转化为利用已知的特征应变和远端剪切应力来求解表征的平衡边界的正应变 $\varepsilon_1$、$\varepsilon_2$ 和切应变 $\gamma_1$、$\gamma_2$,从而确定其对应的应力场。以下仅对平面应变问题进行分析。

## 2 初始应变的确定

由均匀切应力 $t$ 引起的基体内的应力分量为:

$$\sigma_x^c = -t\sin 2\varphi, \sigma_y^c = t\sin 2\varphi, \tau_{xy}^c = t\cos 2\varphi \tag{4}$$

两个复势函数为[3]:

$$\phi_1(z_1) = \frac{\bar{\beta}_1 - \mu_2\bar{\alpha}_1}{\mu_1 - \mu_2} \cdot \frac{1}{\zeta_1}$$

$$\phi_2(z_2) = -\frac{\bar{\beta}_1 - \mu_1\bar{\alpha}_1}{\mu_1 - \mu_2} \cdot \frac{1}{\zeta_2} \tag{5}$$

其中:$\bar{\alpha}_1 = \frac{t}{2}(-a\sin 2\varphi + ib\cos 2\varphi)$,

$$\bar{\beta}_1 = \frac{t}{2}(a\sin 2\varphi + ib\cos 2\varphi)$$

对于理想各向异性材料,$\mu_1 = \alpha + i\beta$,$\mu_2 = -\alpha + i\beta$,$(\alpha > 0, \beta > 0)$ 是表征材料正交各向异性程度的两个复参数。由两复函数 $\phi_1$、$\phi_2$ 表示的面内应力分量为[8]

$$\sigma_x(x,y) = 2\mathrm{Re}[\mu_1^2\phi'_1(z_1) + \mu_2^2\phi'_2(z_2)]$$

$$\sigma_y(x,y) = 2\mathrm{Re}[\phi'_1(z_1) + \phi'_2(z_2)]$$

$$\tau_{xy}(x,y) = -2\mathrm{Re}[\mu_1\phi'_1(z_1) + \mu_2\phi'_2(z_2)] \tag{6}$$

面内位移分量可表示为

$$u(x,y) = 2\mathrm{Re}[p_1\phi_1(z_1) + p_2\phi_2(z_2)]$$

$$v(x,y) = 2\mathrm{Re}[q_1\phi_1(z_1) + q_2\phi_2(z_2)] \tag{7}$$

式中:

$$p_1 = c_{11} + ic_{12}\alpha, \quad p_2 = c_{11} - ic_{12}\alpha,$$

$$q_1 = c_{21}\alpha + ic_{22}, \quad q_2 = -c_{21}\alpha + ic_{22}$$

其中:

$$c_{11} = \beta_{12} + (\alpha^2 - \beta^2)\beta_{11}, \quad c_{12} = 2\beta\beta_{11},$$

$$c_{21} = \beta_{12} + \frac{\beta_{22}}{\alpha^2 + \beta^2}, \quad c_{22} = \beta(\beta_{12} - \frac{\beta_{22}}{\alpha^2 + \beta^2})$$

系数 $\beta_{11} = a_{11} - a_{13}^2/a_{33}$,$\beta_{12} = a_{12} - a_{13}^2/a_{33}$,$\beta_{66} = 2(\beta_{11} - \beta_{12}) = 2(a_{11} - a_{12}) = a_{66}$。其中,柔度系数 $a_{ij}$ 可用工程材料常数确定,即 $a_{11} = 1/E_1$,$a_{33} = 1/E_3$,$a_{12} = -v_1/E_1$,$a_{13} = -v_3/E_3$,$a_{44} = 1/G_3$,$a_{66} = 2(a_{11} - a_{12}) = 2(1 + v_1)/E_1 = 1/G_1$。

在平面问题复变函数解法中,椭圆孔边界上有 $\zeta_1 = \zeta_2 = \sigma = e^{i\theta}$,且 $z_1 = x + \mu_1 y$,$z_2 = x + \mu_2 y$,代入(6)式得

$$\phi_1(z_1) = \frac{t}{4\alpha}(m_1 + in_1)e^{-i\theta}$$

$$\phi_2(z_2) = -\frac{t}{4\alpha}(m_2 + in_2)e^{-i\theta} \tag{8}$$

其中:

$$m_1 = a\cos 2\varphi - a\alpha\sin 2\varphi + b\beta\cos 2\varphi$$

$$n_1 = b\sin 2\varphi + b\alpha\cos 2\varphi + a\beta\sin 2\varphi$$

$$m_2 = a\cos 2\varphi + a\alpha\sin 2\varphi + b\beta\cos 2\varphi$$

$$n_2 = b\sin 2\varphi - b\alpha\cos 2\varphi + a\beta\sin 2\varphi$$

将(8)式代入(7)式并化简为(1)式的形式,得初始应变为:

$$\varepsilon_a^c = t[-c_{11} - c_{12}(R + \beta)]\sin 2\varphi$$

$$\varepsilon_b^c = t[c_{21}(1 + R\beta) - c_{22}R]\cos 2\varphi$$

$$\gamma_a^c = t[-c_{11} - c_{12}(\frac{1}{R} + \beta)]\cos 2\varphi$$

$$\gamma_b^c = t[-c_{21}(1 + \frac{\beta}{R}) + c_{22}\frac{1}{R}]\sin 2\varphi \tag{9}$$

## 3 夹杂-基体系统的应变能

设椭圆夹杂受到非弹性均匀特征应变引起的位移为

$$u^0 = \varepsilon_a^0 x + \gamma_a^0 y, \quad v^c = \varepsilon_b^0 y + \gamma_b^0 x \tag{10}$$

若没有基体存在,则椭圆异质体可在特征应变的作用下达到自由变形边界,如图2所示。但是由于基体的存在并限制,椭圆最终也到达平衡边界。则椭圆夹杂内的弹性应变为

$$\varepsilon_x^0 = \varepsilon_1 - \varepsilon_a^0, \quad \varepsilon_y^0 = \varepsilon_2 - \varepsilon_b^0$$

$$\gamma_{xy}^{0} = \gamma_1 + \gamma_2 - \gamma_a^0 - \gamma_b^0 \qquad (11)$$

其中 $\varepsilon_x^0$、$\varepsilon_y^0$、$\gamma_{xy}^0$ 为弹性应变,与夹杂有关的物理量加注上标“0”。

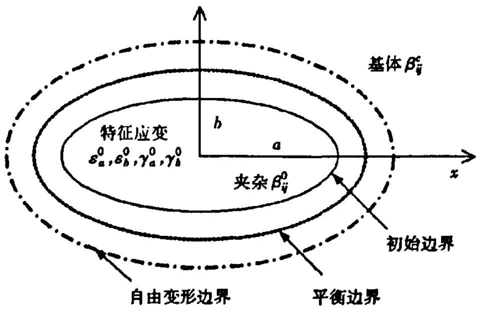


**图2　特征应变作用下的椭圆夹杂变形示意图**

**Fig. 2　Schematic of an elliptical inhomogeneity with eigenstrains**

对于平面应变问题,夹杂内的弹性应变能为

$$W_I = \frac{\pi ab}{2[\beta_{11}^0\beta_{22}^0 - (\beta_{12}^0)^2]}[\beta_{22}^0(\varepsilon_1 - \varepsilon_a^0)^2 - 2\beta_{12}^0(\varepsilon_1 - \varepsilon_a^0)(\varepsilon_2 - \varepsilon_b^0) + \beta_{11}^0(\varepsilon_2 - \varepsilon_b^0)^2] + \frac{1}{\beta_{66}^0}(\gamma_1 + \gamma_2 - \gamma_a^0 - \gamma_b^0)^2 \qquad (12)$$

对于基体内应变能,引入如下的保角变换

$$z = \omega(\zeta) = \frac{a+b}{2\zeta} + \frac{(a-b)\zeta}{2} \qquad (13)$$

将椭圆夹杂外的平面区域变换成一个单位圆面内的区域 $|\zeta|<1$,在单位圆边界上 $\zeta = \sigma = e^{i\theta}$,显然,$\sigma$ 的共轭复数 $\bar{\sigma} = 1/\sigma$。复变量 $z = x + yi$ 经过变换后得到其实部和虚部分别为 $x = a\cos\theta$,$y = -b\sin\theta$。分别引入复函数 $A(\sigma)$,$B(\sigma)$ 依次作为函数 $\phi_1(z_1)$,$\phi_2(z_2)$ 的保角变换后的等价形式,根据(7)式,得平衡边界的位移为

$$\begin{aligned} u(\sigma) &= 2\mathrm{Re}[p_1A(\sigma) + p_2B(\sigma)] \\ v(\sigma) &= 2\mathrm{Re}[q_1A(\sigma) + q_2B(\sigma)] \end{aligned} \qquad (14)$$

利用 Schwartz 公式和 Cauchy 公式求出得到 $A(\sigma)$,$B(\sigma)$ 并代入(6)式可得基体内边界上的应力分量 $\sigma_x^c$、$\sigma_y^c$ 和 $\tau_{xy}^c$ 的结果如下

$$\begin{Bmatrix}\sigma_x^c \\ \sigma_y^c \\ \tau_{xy}^c\end{Bmatrix} = \frac{1}{F(\theta)}\begin{pmatrix} P_1(\theta) & P_2(\theta) & P_3(\theta) & P_4(\theta) \\ Q_1(\theta) & Q_2(\theta) & Q_3(\theta) & Q_4(\theta) \\ T_1(\theta) & T_2(\theta) & T_3(\theta) & T_4(\theta)\end{pmatrix}\begin{Bmatrix} D_0 \\ E_0 \\ I_1 \\ I_2\end{Bmatrix} \qquad (15)$$

其中 $F(\theta)$、$P_i(\theta)$、$Q_i(\theta)$ 和 $T_i(\theta)$ $(i=1,2,3,4)$ 的表达式这里从略。

根据 Clapeyron's 理论,基体的应变能为

$$W_M = \frac{\pi a^2}{e}(m_{11}D_0D_0 + m_{12}D_0E_0 + m_{22}E_0E_0 + m_{33}I_1I_1 + m_{34}I_1I_2 + m_{44}I_2I_2) \qquad (16)$$

其中:

$$m_{11} = -\beta(\alpha^2 + \beta^2)\beta_{11}$$

$$m_{12} = -R[\beta_{12} + (\alpha^2 + \beta^2)\beta_{11}]$$

$$m_{22} = -R^2\beta\beta_{11}$$

$$m_{33} = -R^2\beta(\alpha^2 + \beta^2)\beta_{11}$$

$$m_{34} = R[\beta_{12} + (\alpha^2 + \beta^2)\beta_{11}]$$

$$m_{44} = -\beta\beta_{11}$$

## 4　待定参数的确定

椭圆夹杂和基体系统总的应变能为两部分之和,由(12)式和(16)式得

$$W(\varepsilon_1, \varepsilon_2, \gamma_1, \gamma_2) = W_I + W_M \qquad (17)$$

根据最小势能原理

$$\frac{\partial W}{\partial \varepsilon_1} = \frac{\partial W}{\partial \varepsilon_2} = \frac{\partial W}{\partial \gamma_1} = \frac{\partial W}{\partial \gamma_2} = 0 \qquad (18)$$

可以确定出4个平衡边界对应的应变 $\varepsilon_1$、$\varepsilon_2$、$\gamma_1$、$\gamma_2$ 的解析表达式如下

$$\varepsilon_1 = \frac{1}{M_1}(\lambda_{11}\varepsilon_a^0 + \lambda_{12}\varepsilon_b^0 + \lambda_{13}\varepsilon_a^c + \lambda_{14}\varepsilon_b^c)$$

$$\varepsilon_2 = \frac{1}{M_1}(\lambda_{21}\varepsilon_a^0 + \lambda_{22}\varepsilon_b^0 + \lambda_{23}\varepsilon_a^c + \lambda_{24}\varepsilon_b^c)$$

$$\gamma_1 = \frac{1}{M_2}(\lambda_{31}\gamma_a^0 + \lambda_{32}\gamma_b^0 + \lambda_{33}\gamma_a^c + \lambda_{34}\gamma_b^c)$$

$$\gamma_2 = \frac{1}{M_2}(\lambda_{41}\gamma_a^0 + \lambda_{42}\gamma_b^0 + \lambda_{43}\gamma_a^c + \lambda_{44}\gamma_b^c) \qquad (19)$$

其中:

$$M_1 = (2m_{11} + RK\beta_{22}^0)(2m_{22} + RK\beta_{11}^0) - (m_{12} + RK\beta_{12}^0)^2$$

$$M_2 = \left(2m_{33} + \frac{R}{\beta_{66}^0}\right)\left(2m_{44} + \frac{R}{\beta_{66}^0}\right) - \left(m_{34} + \frac{R}{\beta_{66}^0}\right)^2$$

$$\lambda_{11} = RK(2m_{22}\beta_{22}^0 + m_{12}\beta_{12}^0 + R)$$

$$\lambda_{12} = RK(-2m_{22}\beta_{12}^0 - m_{12}\beta_{11}^0)$$

$$\lambda_{13} = 4m_{11}m_{22} - m_{12}^2 + RK(2m_{11}\beta_{11}^0 + m_{12}\beta_{12}^0)$$

$$\lambda_{14} = RK(m_{12}\beta_{11}^0 + 2m_{22}\beta_{12}^0)$$

$$\lambda_{21} = RK(-2m_{11}\beta_{12}^0 - m_{12}\beta_{22}^0)$$

$$\lambda_{22} = RK(2m_{11}\beta_{11}^0 + m_{12}\beta_{12}^0 + R)$$

$$\lambda_{23} = RK(m_{12}\beta_{22}^0 + 2m_{11}\beta_{12}^0)$$

$$\lambda_{24} = 4m_{11}m_{22} - m_{12}^2 + RK(2m_{22}\beta_{22}^0 + m_{12}\beta_{12}^0)$$

$$\lambda_{31} = \frac{R}{\beta_{66}^0}(2m_{44} - m_{34})$$

$$\lambda_{32}=\frac{R}{\beta_{66}^0}(2m_{44}-m_{34})$$

$$\lambda_{33}=4m_{33}m_{44}-m_{34}^2+\frac{R}{\beta_{66}^0}(2m_{33}-m_{34})$$

$$\lambda_{34}=\frac{R}{\beta_{66}^0}(m_{34}-2m_{44})$$

$$\lambda_{41}=\frac{R}{\beta_{66}^0}(2m_{33}-m_{34})$$

$$\lambda_{42}=\frac{R}{\beta_{66}^0}(2m_{33}-m_{34})$$

$$\lambda_{43}=\frac{R}{\beta_{66}^0}(m_{34}-2m_{33})$$

$$\lambda_{44}=4m_{33}m_{44}-m_{34}^2+\frac{R}{\beta_{66}^0}(2m_{44}-m_{34})$$

## 5 应力连续性条件的验证和一些特殊结果

在得出待定系数的过程中,我们没有使用应力连续的边界条件,但是通过利用保角变换和(15)式可进一步确定基体内边界上的应力分布。特别地,在椭圆夹杂边界上,夹杂与基体的法向正应力和切应力分别记为 $\sigma_n^0$、$\tau_{ns}^0$、$\sigma_n^c$、$\tau_{ns}^c$,根据应力转换关系

$$\cos 2\psi=\frac{-1+R^2+(1+R^2)\cos 2\theta}{1+R^2+(-1+R^2)\cos 2\theta}$$

$$\sin 2\psi=\frac{-2R\sin 2\theta}{1+R^2+(-1+R^2)\cos 2\theta} \tag{20}$$

其中 $\psi$ 为边界曲线外法线与 $x$ 轴的夹角。可得夹杂与基体在边界上的法向正应力和切应力相等,基体内边界应力的连续条件完全满足,表明了结果的封闭性。表达式如下:

$$\sigma_n^0=\sigma_n^c=\sigma_n=\sigma_1(\theta)(\varepsilon_a^0-\varepsilon_a^c)+\sigma_2(\theta)(\varepsilon_b^0-\varepsilon_b^c)+\sigma_3(\theta)(\gamma_a^0+\gamma_b^0-\gamma_a^c-\gamma_b^c)$$

$$\tau_{ns}^0=\tau_{ns}^c=\tau_{ns}=\tau_1(\theta)(\varepsilon_a^0-\varepsilon_a^c)+\tau_2(\theta)(\varepsilon_b^0-\varepsilon_b^c)+\tau_3(\theta)(\gamma_a^0+\gamma_b^0-\gamma_a^c-\gamma_b^c) \tag{21}$$

其中:

$$S(\theta)=1+R^2+(-1+R^2)\cos 2\theta$$

$$\sigma_1(\theta)=\frac{2KR^2}{eM_1S(\theta)}\left\{\left[R^2\beta_{22}^0+2R\beta_{11}\beta(\alpha^2+\beta^2)\right]\cos^2\theta+\left[-\beta_{12}^0+\beta_{12}+\beta_{11}(\alpha^2+\beta^2)\right]\sin^2\theta\right\}$$

$$\sigma_2(\theta)=\frac{2KR^2}{eM_1S(\theta)}\left\{R^2\left[-\beta_{12}^0+\beta_{12}+\beta_{11}(\alpha^2+\beta^2)\right]\cos^2\theta+(\beta_{11}^0+2R\beta_{11}\beta)\sin^2\theta\right\}$$

$$\sigma_3(\theta)=\frac{-4R^3\sin\theta\cos\theta}{eM_2S(\theta)\beta_{66}^0}$$

$$\tau_1(\theta)=\frac{KR^2\sin 2\theta}{eM_1S(\theta)}\left\{2\beta_{11}\beta(\alpha^2+\beta^2)+R\left[\beta_{12}^0+\beta_{22}^0-\beta_{12}-\beta_{11}(\alpha^2+\beta^2)\right]\right\}$$

$$\tau_2(\theta)=\frac{KR^3\sin 2\theta}{eM_1S(\theta)}\left\{-2R\beta_{11}\beta+\left[-\beta_{12}^0-\beta_{11}^0+\beta_{12}+\beta_{11}(\alpha^2+\beta^2)\right]\right\}$$

$$\tau_3(\theta)=\frac{R^2}{eM_2S(\theta)\beta_{66}^0}\left[-1+R^2+(1+R^2)\cos 2\theta\right]$$

以上结果均可简化为已知的经典结论。例如对于各向同性材料,取 $\alpha=0,\beta=1$。当夹杂内受均匀特征应变的作用,在平面应力情况下,式(16)可化简为 $W=W_M^1+W_M^2$,其中 $W_M^1$、$W_M^2$ 分别为正应变和切应变表示的应变能,其表达式为:

$$W_M^1=-\frac{\pi a^2}{e}\left[\beta_{11}\varepsilon_1^2+R(\beta_2+\beta_{11})\varepsilon_1\varepsilon_2+R^2\beta_{11}\varepsilon_2^2\right]$$

$$W_M^2=-\frac{\pi a^2}{e}\left[R^2\beta_{11}\gamma_1^2-R(\beta_{12}+\beta_{11})\gamma_1\gamma_2+\beta_{11}\gamma_2^2\right] \tag{22}$$

式中,$e=\beta_{12}^2+2\beta_{11}\beta_{12}-3\beta_{11}^2$,$\beta_{11}=\frac{1-v^2}{E}$,$\beta_{12}=\frac{-v(1+v)}{E}$,$E$,$v$ 分别为材料弹性模量和泊松比。上面结果即为经典结果[9]。此外对于各向同性材料仅受外载作用的情形,式(9)可退化为经典结果[10]。

## 6 小结

本文给出了远端剪切作用下,一类正交各向异性介质中平面椭圆夹杂受非弹性均匀特征应变引起的应力场的封闭形式的解析解。通过检验边界上法向正应力和切应力的连续条件表明了结果的正确性,并且本文结果可退化到已有的经典结果。本文的解可做为正交各向异性材料在特征应变和外部均匀剪切共同作用下的材料疲劳失效与断裂损伤的理论分析依据。

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# Electronic and Magnetic Properties of Semifluorinated Graphene Sheet

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**Abstract**: The electronic and magnetic properties of the semifluorinated graphene sheet are investigated by density functional theory. The results show that semifluorination makes graphene sheet as a antiferromagnetic semiconductor with an indirect band gap of 1.00 eV. Due to attractive interaction of fluorine atoms with fluorinated carbon atoms, the fluorinated carbon atoms and unfluorinated carbon atoms belong to two different planes and the distance between the planes is 0.286 Å, the length of carbon - carbon bond reaching 1.501 Å. So, semifluorination translates the nonmagnetic material into antiferromagnetic semiconductor with band gap. And it indicates that it may have potential applications for future functional nanodevices.

**Keywords**: density functional theory; semifluorination; antiferromagnetic semiconductor

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# Analysis of Elastic Fields in an Elliptic Inhomongeneity in Orthotropic Media under Uniform Shear at Infinity

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**Abstract**: This paper presents an analytical solution for the elastic fields induced by non - elastic eigenstrains in a plane elliptical inhomogeneity embedded in the orthotropic matrix under shear at infinity and inclined at any angle. The conformal transformation and complex function method for anisotropic elastic material were used to transform the shear to the "initial strains" in the matrix and to generate a closed - form solution for the elastic strains/stress fields in the inhomogeneity and matrix in terms of the principle of the minimum potential energy.

**Keywords**: non - elastic eigenstrains; orthotropic; conformal transformation; elliptic inhomongeneiy

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